

START - ALL PARTICIPANTS

1 – Pets (coefficient 1)

Alan, Beatrice, Cindy and Damien each own a different one of the following pets: a dog, a cat, a parrot or a hamster.

Alan says: "I don't own a parrot or a hamster".

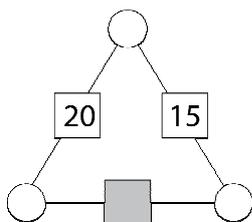
Beatrice says: "I don't own a dog or a hamster".

Cindy says: "My cat is called Shadow".

Who owns which pet?

2 – Triangle of the Year (coefficient 2)

The three circles at the points of the triangle in the drawing each contain a whole number greater than 1. The numbers at the ends of each side are multiplied together and the result is written in the square at the middle of the side.



Four numbers are not shown on the drawing.

What is the number in the grey square?

3 – Palindrome Time (coefficient 3)

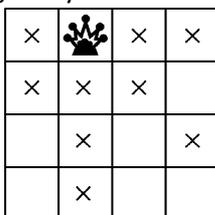
It's 15:51, a palindrome time (a time which can be read from left to right or from right to left).

How many minutes must one wait until the next palindrome time?

4 – The Chess Queen (coefficient 4)

In a game of chess, a queen can move any number of squares horizontally, vertically or diagonally.

So, on the mini-chessboard drawn here, the queen can move to one of the nine squares marked with a cross, not counting its current square.



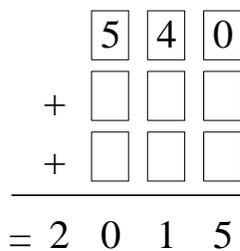
Matthew places a queen on a square of a normal chessboard of 64 squares.

What is the maximum number of squares to which the queen could move, not counting its current square?

5 – The Addition (coefficient 5)

In this addition, the nine rectangles all contain different single digits.

In each column, the digits of the numbers to be added in the rectangles from the top to the bottom are ordered from smallest to largest.



Complete the addition.

6 – Grace's Case (coefficient 6)

Grace transports three types of precious stones in her case. In total, there are more than 70 stones.

There are three times as many emeralds as rubies, and a quarter as many emeralds as diamonds.

In Grace's case there are fewer than 70 diamonds. How many emeralds are there? ?

7 – Not enough space (coefficient 7)

Matilda's calculator displays the digits 0 to 9 as in the drawing below.



But, following a fault, the machine no longer shows any space between the digits, which thus seem to be stuck together.

Matilda has entered a number of 7 different digits (which does not contain the digit 1) and her calculator displays :



What is the smallest possible number this could be?

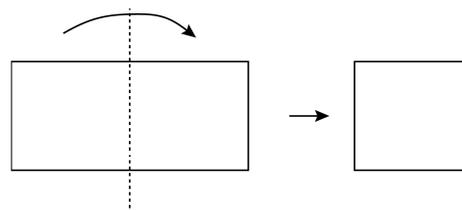
8 – Valentine's Day (coefficient 8)

Valentine's day this year was 14.02.2015. The sum of the digits of this date is 15. **How many other dates in 2015 also have digits that sum to 15?**

END FOR CM PARTICIPANTS

Problems 9 to 18 : beware! For a problem to be completely solved, you must give both the number of solutions, AND give the solution if there is only one, or give two solutions if there are more than one. For all problems that may have more than one solution, there is space for two answers on the answer sheet (but there may still be a unique solution).

9 – The Folded Rectangle (coefficient 9)



A non-square rectangle of paper has sides measuring whole numbers of centimetres. It is folded along the line of symmetry perpendicular to the longer two sides as indicated in the figure, which is not to scale. The rectangle thus obtained has a perimeter of 15 cm.

What is the perimeter of the original rectangle?

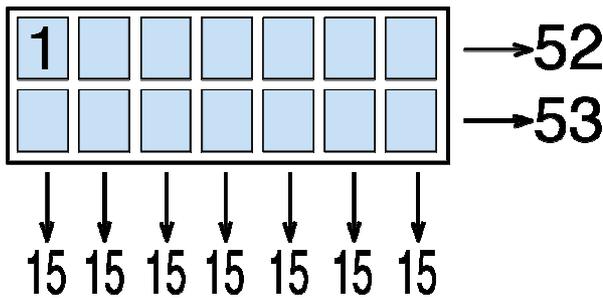
10 – Laps of the Track (coefficient 10)

Two runners are running laps around a circular running track, each at constant speed. They leave simultaneously from the start line, and pass this line simultaneously for the first time after six minutes. The faster runner laps at 5 seconds faster than the slower.

How many seconds does the faster runner take to run a lap?

END FOR CE PARTICIPANTS

11 – The 14 Numbers (coefficient 11)



Matilda wants to place the integers from 1 to 14 in the rectangle above, one number per small box (the 1 is already placed), so that she obtains the sums shown by each row and column, and so that the numbers in the boxes on the upper row are in order (smallest on the left).

Complete the rectangle.

On the answer sheet, only write the 7 numbers of the upper line.

END FOR C1 PARTICIPANTS

12 – Octo-pass (coefficient 12)

8 points are placed on a circle to form a regular octagon. On the points, in order around the circle, are written the numbers 0, 2, 0, 5, 2, 0, 1, 5.

Starting from a number 0, without lifting the pencil a path of seven different straight lines is drawn, the numbers on the path indicating today's date 0-2-0-5-2-0-1-5. The pattern of drawn lines contains six angles between the adjacent lines.

What is the minimum sum of these six angles?

13 – Röstigraben (coefficient 13)

21 adults and 47 children are crossing the Sarine river. Their boat cannot hold more than 100 kg total weight. Each child weighs 40 kg, each adult 80 kg.

How many crossings is the minimum necessary to allow everyone to cross?

Beware! To cross the river and return to the first side counts as two crossings. There always has to be at least one person on the boat for a crossing.

14 – Two products for a sum (coefficient 14)

Mathew has found a pair of irreducible fractions whose numerators and denominators are digits greater than 4. These two fractions have the remarkable property that their sum is equal to twice their product.

What are these two fractions?

Note: The order of the pair does not matter, for example 2 and 3 is the same pair as 3 and 2.

END FOR C2 PARTICIPANTS

15 – Pandora's Box (coefficient 15)

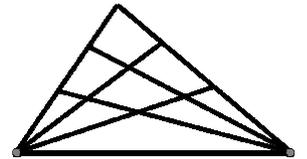
Pandora has a box in the shape of a rectangular parallelepiped. She notes that the centres of the three faces which touch the corner of the box where she is touching with her fingertip form a triangle of sides 8, 10 et 12 cm.

What is the volume of the box, rounded to the nearest whole mm³?

Note: the following approximations may be useful $\sqrt{2} = 1.414$; $\sqrt{3} = 1.732$; $\sqrt{5} = 2.236$.

16 – Fan-tastic (coefficient 16)

The drawing represents two fans each of four ribs that cross each other.



How many triangles would be formed by two fans crossing, each with 2015 ribs?

END FOR L1 AND GP PARTICIPANTS

17 – Champagne! (coefficient 17)

A champagne bowl is in the form of a hemisphere of radius 9 cm. The bowl is filled to the brim, then a rectangular parallelepiped of size a x b x 10 cm is partially submerged in it.

What are the values of a and b such that the minimum amount of champagne remains in the glass?

Give the answers in mm rounded to the nearest 0.1 mm if necessary.

Note: the following approximations may be useful $\sqrt{2} = 1.414$; $\sqrt{3} = 1.732$; $\pi = 3.1416$.

18 – Square palindrome (coefficient 18)

The smallest palindromic square number bigger than 1 with an odd number of digits is $121 = 11^2$, and the next smallest is $484 = 22^2$.

The smallest palindromic square number with an even number of digits is $698\ 896 = 836^2$.

What is the next smallest?

END FOR L2 AND HC PARTICIPANTS



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Information and results on <http://fsjm.ch/>