

Semi-finals FFJM - March 23th, 2013

Information and rankings on <http://fsjm.ch>

START - ALL PARTICIPANTS

1 – DEDUCTION (Coefficient 1)

After a skateboarding accident, Inspector Dede Duction interrogates the three people on the scene.

Anatole says: "Camille told the truth".

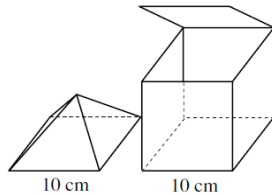
Bea says: "Camille lied".

Camille says: "Anatole told the truth".

The inspector knows that only one of the three lies all the time, while the two others always tell the truth. **Who is the liar?**

2 – THE PYRAMIDS (Coefficient 2)

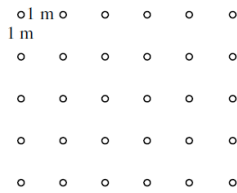
Mathilde found in her grandfather's attic ten wooden Pyramids with a square base of side length 10 cm. These pyramids all have the same height, 5 cm.



Mathilde has a box shaped like a cube of side length 10 cm. She stores a certain number of the pyramids in the box and closes the lid. **How many pyramids could she store in the box at most?**

3 – THE SHEEP (Coefficient 3)

Mathias' father has 14 fence segments, each one meter long, that he wants to use to create an enclosed pasture for his sheep. Each fence must be placed between two vertices of the grid



depicted that are distant one meter from each other. Each sheep must have access to an amount of pasture corresponding to the area of a square in the grid. **How many sheep at most can Mathias' father keep in his pasture?**

4 – BROKEN CALCULATOR (Coefficient 4)

On this calculator, only three keys still work: +, x, and 5. When the "5" key is pressed, the calculator automatically computes and displays the result of the last operation.

The calculator initially displays 5. After having pressed four times one of "+" or "x" and then "5", the calculator displays 100.

What are the three intermediate results displayed by the calculator?

5 – SELF-REFERENCE (Coefficient 5)

23.03.2013

In this frame, there are:

... digits 0; ... digits 1;

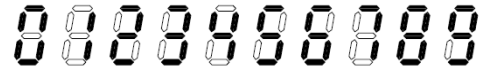
... digits 2; ... digits 3;

Complete the dotted lines by writing four digits into the frame. The sentence in the frame must then be correct. Beware, all digits written in the frame must be taken into account!

END FOR CE PARTICIPANTS

6 – THE CLOCK (Coefficient 6)

On this clock, the digits from 0 to 9 are displayed as shown on the figure. Ever since it broke down, the clock



only displays one single digit, and every second, a single display segment changes either from lit to unlit or the other way around.

The clock initially displays "0". After 5 seconds it has successively shown 6 different digits, including the initial "0".

What is the last digit displayed?

7 – THE NINE DIGITS (Coefficient 7)

This multiplication uses the nine digits from 1 to 9, each once.

$$\begin{array}{r} _ _ _ 8 \\ \times 4 \\ \hline = _ _ _ _ \end{array}$$

Fill in the blanks!

8 – THE SUM OF THE YEAR (Coefficient 8)

Mathias writes all four-digit numbers that can be written with one 2, one 0, one 1 and one 3. He then adds all these numbers. **What result does he get?**

END FOR CM PARTICIPANTS

Problems 9 to 18: beware! For a problem to be completely solved, you must give the number of solutions, AND give the solution if there is only one, or two solutions if there is more than one. For all problems that may admit more than one solution, there is space for two answers on the answer sheet (but there may still be a unique solution).

9 – THE TWO VILLAGES (Coefficient 9)

Alphavillage and Betabled are two villages 10 km apart from each other. Several highways pass near these two villages, all of them going in straight lines. Each of them passes at its closest point exactly 2 km from Alphavillage and exactly 3 km from Betabled.

How many such highways are there at most?

10 – PLANET KRYPTON (Coefficient 10)

On planet Krypton, 1/4 of the total population is left-handed, and 1/5 of the others are right-handed; the remaining Kryptonians are perfectly ambidextrous.

What percentage of the total population is right-handed?

(Note: none of the left-handed and right-handed inhabitants is ambidextrous)

11 – DOUBLE LOSS (Coefficient 11)

Three players played three rounds of "double loss". In each round, there is one loser, who must pay the other players so as to double the holdings of each of them. (The game ends if the loser is unable to do so).

After the three rounds, each player has 24 euros.

What were the initial holdings of each player before the game started, in increasing order, given that none of them had more than 40 euros?

END FOR C1 PARTICIPANTS

12 – BY 5 AND BY 6 (Coefficient 12)

The sum of the digits of 987 is divisible by 6: $9+8+7 = 6 \times 4$.

The sum of the digits of 988 is divisible by 5: $9+8+8 = 5 \times 5$.

What are the two smallest consecutive positive integers so that the sum of the digits of the smaller one is divisible by 6 and the sum of the digits of the greater one is divisible by 5?

13 – MATHILDE'S TRIANGLES (Coefficient 13)

Mathilde draws all the right triangles which satisfy the following conditions:

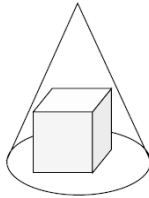
- the lengths of the edges adjacent to the right angle, expressed in millimeters, are integers;
- the area is equal to 2013 mm^2

How many such triangles are there?

Consider two triangles identical if they agree after rotation and reflection.

14 – MAGE HIC'S TRICK (Coefficient 14)

Mage Hic's hat is a circular cone whose base is a disc with a radius of 27 cm, and with a height of 70 cm. After having uttered the magic formula, Mage Hic makes a cube appear inside the cone.



What is the edge length of the cube, at most?

Take 1.414 for square root of 2 if needed, and round to the nearest millimeter.

END FOR C2 PARTICIPANTS

15 – THE SQUARE AROUND THE TRIANGLE (Coefficient 15)

An equilateral triangle with an edge length of 10 cm is drawn on Mathias' notebook. He then draws a square whose edges pass through the three vertices of the triangle.

What is the minimum possible length of the edge of the square?

Take 1.414 for square root of 2 and 1.732 for square root of 3 if necessary, and round the answer in cm to the nearest 1/100.

16 – THE TWO NUMBERS (Coefficient 16)

Two positive integers (different from zero) are such that:

- the difference between their squares is a perfect cube;
- the difference between their cubes is a perfect square.

What is the largest of the two numbers, given that it's less than 20?

END FOR L1 AND GP PARTICIPANTS

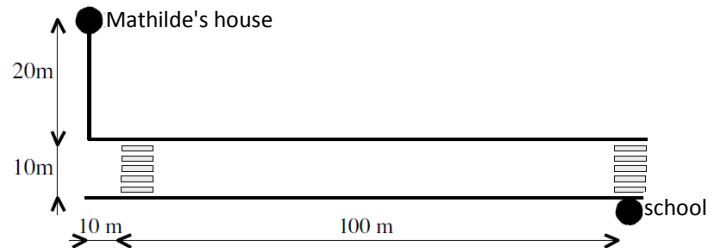


17 – A VERY PRODUCTIVE YEAR (Coefficient 17)

The number 2013 is productive: the products of consecutive digits, 2×0 , 0×1 and 1×3 , namely 0, 0 and 3, can all be read within 2013. Similarly for 1261, since 2, 12, and 6 can all be read within 1261.

What is the smallest productive number that is written using all ten digits from 0 to 9?

18 – THE WAY TO SCHOOL (Coefficient 18)



When she goes to school, Mathilde always walks at 4.5 km/h. Mathilde has devised a strategy that allows her to leave from home as late as possible while ensuring that she will never get to school later than 8:30. For this, she has noticed that the pedestrian traffic lights at the two crossings on the street that her school is on are green for 15 s then red for 45 s. Moreover they are synchronized (they have the same color at the same time) and can be seen from anywhere on the street. However, the time at which they start in the morning is not fixed, so it is impossible to know ahead of time when they will turn green and Mathilde only discovers the state of the two lights when she gets to the intersection between the street she lives on and that of her school.

What is the average time at which Mathilde gets to school, given that, once out of her home, she always tries to get to school as early as possible?

Round your answer to the nearest second.

END FOR L2 AND HC PARTICIPANTS



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